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Int. J. Heat Mass Transfer. Vol. 13, pp. 215–218. Pergamon Press 1970. Printed in Great Britain

TWO-DIMENSIONAL SOLIDIFICATION IN A CORNER

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(*Received* 19 *March* 1969)

NOMENCLATURE

C_p, specific heat at constant pressure $\left[\frac{\text{Btu}}{\text{lbm°F}}\right]$;
K, thermal conductivity $\left[\frac{\text{Btu}}{\text{min ft°F}}\right]$;
L, latent heat of fusion $\left[\frac{\text{Btu}}{\text{lbm}}\right]$;
T, temperature [°F];
T_i^*, dimensionless initial temperature $\equiv \frac{K_L}{K_s}\frac{T_i - T_F}{T_F - T_W}$;
T_L^*, dimensionless liquid temperature $\equiv \frac{K_L}{K_s}\frac{T_L - T_F}{T_F - T_W}$;
T_s^*, dimensionless solid temperature $\equiv \frac{T_s - T_F}{T_F - T_W}$;
t, time [min];
x, distance [ft];
x^*, dimensionless distance $\equiv \frac{x}{2\sqrt{(\alpha_s t)}}$;
y, distance [ft];
y^*, dimensionless distance $= \frac{y}{2\sqrt{(\alpha_s t)}}$.

Greek symbols
α, thermal diffusivity $= \frac{k}{\rho C_p}\left[\frac{\text{ft}^2}{\text{min}}\right]$;
β, ratio of latent to sensible heat $= \frac{L}{C_{p_s}(T_F - T_W)}$;
ρ, density $\left[\frac{\text{lbm}}{\text{ft}^3}\right]$.

Subscripts
F, freezing condition;
i, initial condition;
L, liquid region;
o, refers to condition on the interface $x = y$;
s, solid region;
W, wall condition.

INTRODUCTION

HEAT conduction in systems undergoing phase transformation is encountered in numerous engineering systems. Examples are freezing, melting, ablation, welding and casting. Much of the research in this field has been theoretical in nature, based on one-dimensional models. Limited theoretical, as well as experimental, work has been done on two-dimensional systems [1–4].

This note presents experimental findings of two-dimensional solidification of liquid in a corner. Comparisons are made with an analytical solution to this problem which was obtained in [1]. The problem considered in [1] is that of freezing or melting of a liquid, initially at a uniform temperature and filling the quarter-space $x, y > 0$, subject to a constant wall temperature.

EXPERIMENTAL APPARATUS

The apparatus consisted of a 12 in. square container, 2 in. deep with a 2 in. brass channel fitted along two sides. Polyurethane foam was used to insulate the four sides of the square. Air gaps 1 in. deep were provided to insulate the two other sides of the container. The system is illustrated in Fig. 1.

Temperature measurements were made with a grid of 37 chromel–alumel thermocouples. Thermocouple locations were arranged to check symmetry and interface location. A Sanborn recorder was used to monitor thermocouple readings. The accuracy of temperature readings is ±0·1°F.

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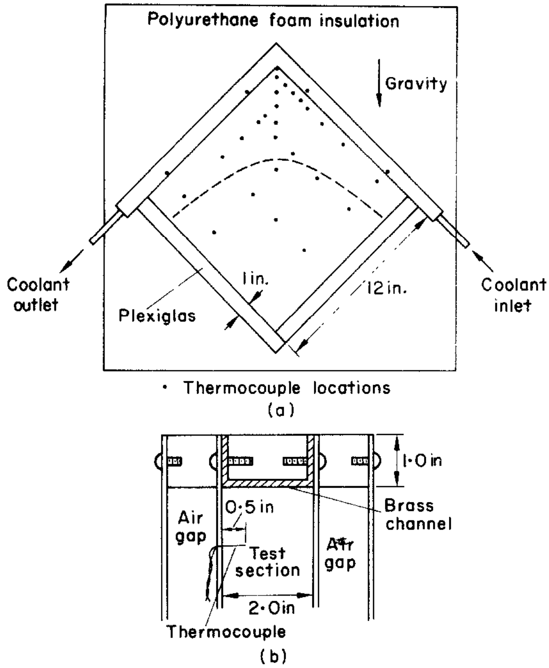

FIG. 1. Experimental apparatus: (a) Top view showing thermocouple grid (b) Cross-section of apparatus showing thermocouple placement.

A tracing technique was used to measure the location of solid–liquid interface. The method is estimated to be accurate to within $\pm 0{\cdot}04$ in. Photographs were also used to determine interface transients.

To maintain constant wall temperature, refrigerated ethylene glycol was circulated through the brass channel. The wall transient was measured by embedding thermocouples on the inside and outside surfaces of the channel. The outside surface temperature was observed to reach the coolant temperature in approximately 40–80s. Due to limitation of the refrigeration system the lowest wall temperature achieved was 14°F. Experiments were thus limited to materials with a freezing temperature above 14°F. Distilled and deaerated water was used as the subject material and tests were conducted at various initial temperatures.

Prior to each run the system was refrigerated until the desired initial temperature was achieved. The apparatus was then removed from the refrigerator, thermally insulated and placed on the test stand for three hours to allow the water to settle without introducing temperature gradients in the system.

Due to the density variation of water with temperature and its inversion at 39°F, free convection currents were observed in the system resulting in temperature inversions. This phenomena is strongly dependent on the orientation of the apparatus with respect to the direction of gravity. To retain

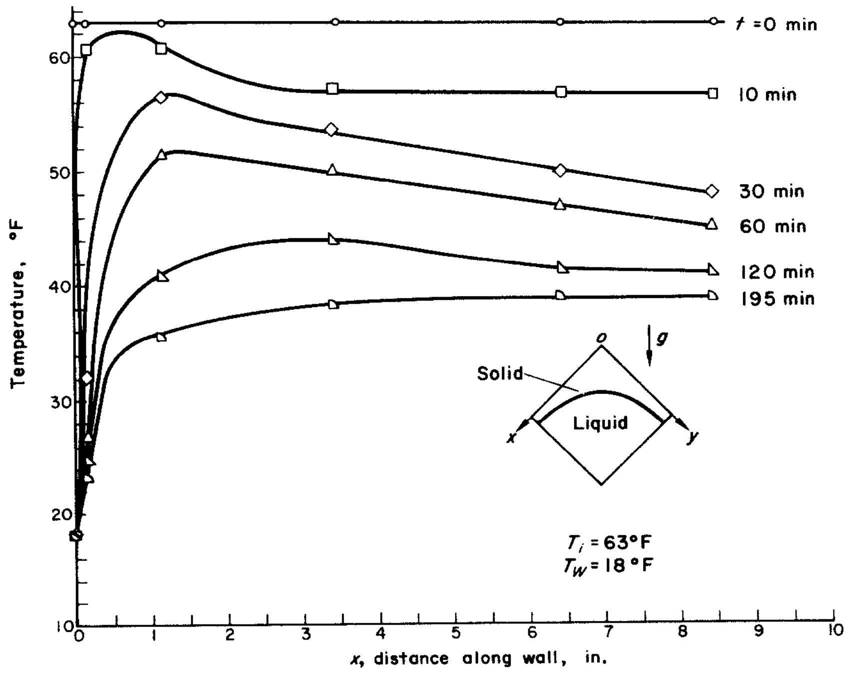

FIG. 2. Transient temperature profiles along $x = y$ showing temperature inversion.

symmetry and minimize three dimensional effect on interface motion, the system was mounted diagonally as shown in Fig. 1. Transient temperature profiles with temperature inversion for a typical test are shown in Fig. 2.

RESULTS

The analytical solution of the two-dimensional solidification problem in a quarter space presented in [1] shows that the problem is characterized by similarity in the variables $x^* = x/2\sqrt{(\alpha_s t)}$ and $y^* = y/2\sqrt{(\alpha_s t)}$. Thus the moving interface becomes stationary in the x^*, y^* plane (Fig. 3). It was

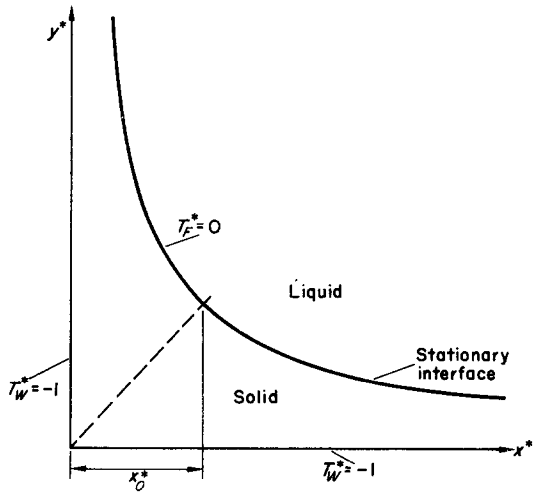

FIG. 3. The non-dimensional stationary problem.

Table 1.

β	T_i^*	x_0 (in.)	t(mm)	x_0^* Experiment	x_0^* Theory
33·9	1·35	0·66	90	0·107	
		0·78	120	0·104	
		1·01	150	0·121	
		1·07	210	0·109	
		1·26	270	0·113	
		1·36	300	0·115	
		1·64	390	0·122	
		1·92	450	0·125	
			Average	0·114	0·122
22·4	0·553	0·31	10	0·143	
		0·55	30	0·146	
		0·75	60	0·142	
		1·00	120	0·134	
		1·27	180	0·139	
			Average	0·141	0·159
19·6	0·25	0·33	10	0·151	
		0·57	30	0·153	
		0·79	60	0·149	
		1·13	120	0·151	
		1·40	180	0·154	
			Average	0·152	0·178

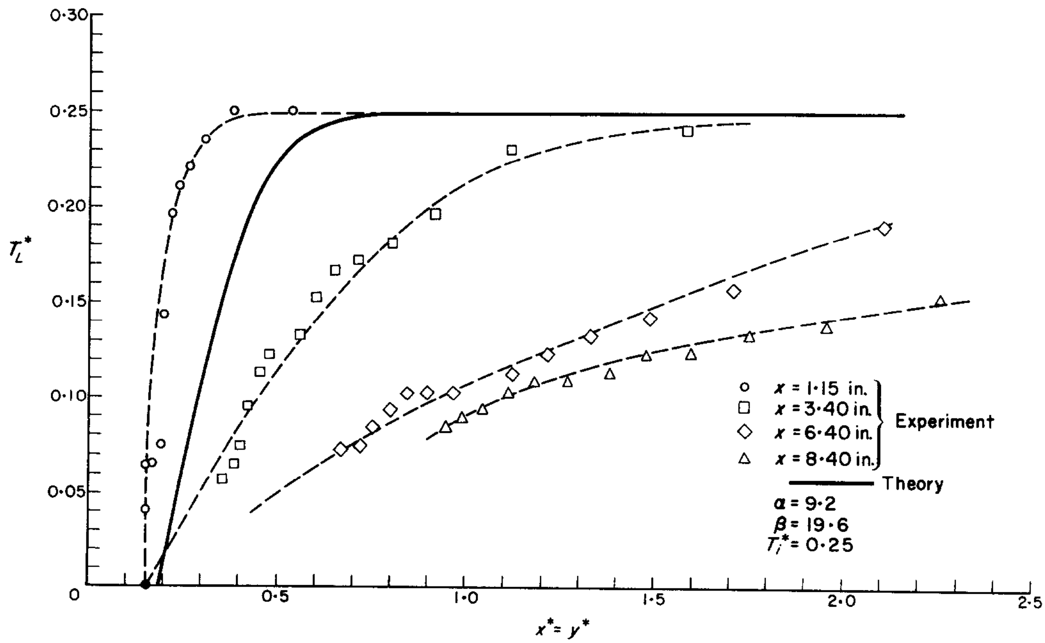

FIG. 4. Liquid temperature distribution along $x^* = y^*$ illustrating non-similarity of profiles due to density inversion.

also shown that interface location and temperature distribution depend on three parameters; $\beta = L/C_{p_s}(T_F - T_W)$, $T_i^* = (K_L/K_s)\ (T_i - T_F)/(T_F - T_W)$ and $\alpha = \alpha_s/\alpha_L$. The dimensionless temperatures in the solid and liquid are; $T_L^* = (K_L/K_s)\ (T_L - T_F)/(T_F - T_W)$ and $T_s^* = (T_s - T_F)/(T_F - T_W)$.

Table 1 gives a comparison between experimental and theoretical values of x_o^*, which is the interface location at $x^* = y^*$ for three different values of β and T_i^*. In each case the experimental value for x_o^* was calculated at different intervals of time. Results indicate that for a given β and T_i^*, x_o^* is nearly constant which confirms the similarity nature of the problem as predicted by theoretical consideration.

Figure 4 gives the liquid temperature transients at the diagonal $x^* = y^*$ for various locations. We observe that the profiles do not coalesce into a single curve as predicted by the theoretical solution. This is a direct consequence of the temperature inversion due to density variations of the liquid.

The one-dimensional behaviour of the system far away from the corner is illustrated in Fig. 5. Temperature measurements at $x = 10{\cdot}4$ in. from the corner are compared with Neumann's one-dimensional solution. Excellent agreement is observed in the solid region.

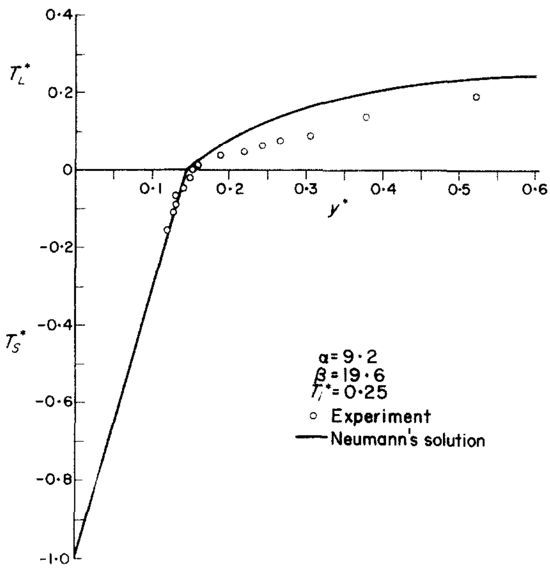

FIG. 5. Comparison between temperature measurements in the one-dimensional region ($x = 10{\cdot}4$ in.) and Neumann's solution.

ACKNOWLEDGEMENT

The support of the National Science Foundation through Grant No. GK 1281 is acknowledged.

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Int J Heat Mass Transfer Vol 13, pp 218–221 Pergamon Press 1970. Printed in Great Britain

TRANSIENT HEATING OF THIN PLATES

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(*Received* 12 *March* 1969 *and in revised form* 4 *August* 1969)

NOMENCLATURE

$E_1(x)$,	exponential integral function;
erf(x),	error function;
erfc(x),	complementary error function;
h,	coefficient of surface heat transfer;
$I_\nu(x)$,	modified Bessel function of the first kind;
$J_\nu(x)$,	Bessel function;
$K_\nu(x)$,	modified Bessel function of the third kind;
k,	thermal conductivity;
Q_0,	maximum flux of heat source;